A STUDY OF BUNCH LENGTH MEASUREMENT AT ARGONNE WAKEFIELD ACCELERATOR

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1. INTRODUCTION

The Argonne Wakefield Accelerator Group is conducting a series of diagnostics to characterize their new, high-charge photocathode drive gun. One significant attribute is the electron bunch length, currently found via a streak camera to measure the light produced by a quartz Cherenkov radiator. Once accurately defined, it is essential to establish an effective means of collecting and transporting the Cherenkov light.

Cherenkov radiation is emitted, mainly in the visible and near-visible regions of the spectrum, by a transparent medium when charged particles travel through it at a velocity greater than the speed of light inside the medium [1]:

$$v > v_c = c/n$$

where:

- n is the refractive index of the medium
- v is the velocity of the charged particle
- v_c is the threshold velocity.

The relativistic electrons traversing the particular medium temporarily polarize the atoms, creating a dipole field along the trajectory axis. The asymmetrical polarizing field set up by the electron causes the atoms to radiate a brief electromagnetic pulse. Because the particle velocity exceeds the threshold velocity of light in the medium, the radiated wavelets are in phase with one another. These spherical wave fronts of radiation are analogous to the creation of a shock wave, or sonic boom, when an object exceeds the speed of sound in a medium. The light is emitted at a fixed angle with respect to the trajectory or track of the charged particle. This is the Cherenkov angle, given by

$$\theta_c = \cos^{-1} (1/ \beta \cdot n(\omega))$$

where:

- $\beta = (v/c)$ is the relativistic factor.
- $n(\omega)$ is the frequency-dependent refraction index of the medium.

Given $\beta = 0.997$ and a reasonable $n(\omega)$ range of 1.5 to 1.6, the resulting Cherenkov angle varies between 48.04° and 51.19°.

2. SETUP

The Cherenkov apparatus is placed in a multi-cross vacuum chamber that will allow the radiation bunch to exit through a window to the residing optics. The quartz radiator is not set at normal incidence to the electron beam axis, but oriented at a specified angle to avoid internal reflection. The resulting angle of incidence is the Brewster's angle, which is calculated with the reflected and refracted rays normal to one another. Using Snell's law, the Brewster's angle θ_B occurs when

$$\theta_B = \tan^{-1} (n_2/n_1).$$

The Brewster angle for $n_1 = 1.5$ and $n_2 = 1.0$ is 33.69°, and 32.01° for $n_2 = 1.6$. An application of Snell's law [2] is also used to explain the observation angle with respect to the beam trajectory:

$$n_1 \sin (\theta_c - \theta_q) = n_2 \sin (\theta_o - \theta_q)$$

where:

- $n1$ is the index of refraction of the optical medium
- $n2$ is the index of refraction of vacuum
- θ_c is the Cherenkov angle
- θ_o is the observation angle
- θ_q is the quartz angle.

The geometry and orientation of the quartz is illustrated in Figure 1. All angles, other than the incident angle θ_i and the refracted angle θ_r, are measured with respect to the trajectory of the charged particle. Table 1 gives the results of several values of n and the angles listed above.

n	θ_B	θ_c	θ_q	θ_o
1.50	33.69	48.04	15.00	69.87
1.55	32.83	49.68	15.00	76.88
1.60	32.01	51.19	15.00	85.85
1.50	33.69	48.04	16.00	68.73
1.55	32.83	49.68	16.00	75.27
1.60	32.01	51.19	16.00	83.22
1.50	33.69	48.04	17.00	67.67
1.55	32.83	49.68	17.00	73.82
1.60	32.01	51.19	17.00	81.03

Table 1.

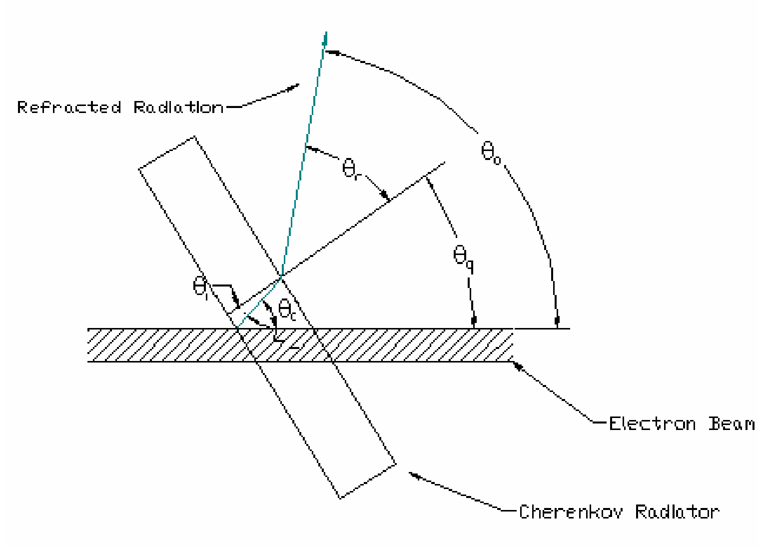

Figure 1. The basic defining angles of Cherenkov radiation.

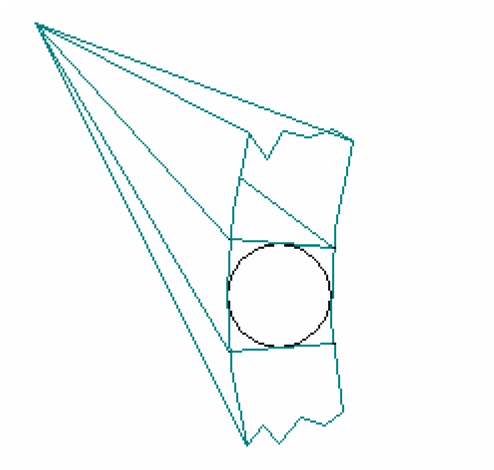

Figure 2. Conical segment of Cherenkov light passing a lens.

Given the slight variability of the observation angle due to the frequency dependence of the dielectric constant of the quartz, the resulting angle of incidence entering the first lens is crucial to the light intensity measured at the streak camera. Original measurements of the optics with a low wattage light bulb and specific aperture allows the first lens to be between 25 to 30 cm away from the quartz and still properly collimate light to the streak camera with minimal aberration or loss of light. By measuring the diameter of the lens and its distance from the Cherenkov radiator, an estimate can be made of the percentage of light actually incident on the lens. With the observation angle ranging from 68.73° to 83.22°, the acceptance angle is about 14.5°. This result puts the maximum distance away with a 7 cm lens at 27.5 cm without any loss of light. Figure 2 illustrates an example of the conical boundaries of the light with respect to the lens. A better-defined dispersion curve of the quartz will allow further refinement of the angle of observation as well as lessening geometric constraints.

A series of lenses and mirrors transports the light out of the accelerator tunnel and into the laser room where a streak camera is used to measure the bunch length. The transport distance is 15.8 m in length. The Hamamatsu M1952/C1587 streak camera is designed to temporally distribute, or streak, the light bunch over a given area where it can be displayed visually. See Figure 3 for a snapshot of the live camera display. The digital picture is then analyzed using dedicated software to produce a distribution of light verses time as seen in Figure 4.The bunch length is found by carefully selecting the full width of the half maximum, FWHM, from the graph provided. The resulting number of pixels, multiplied by a 6:1 picosecond-to-pixel ratio, gives a bunch length of approximately 10 ps.

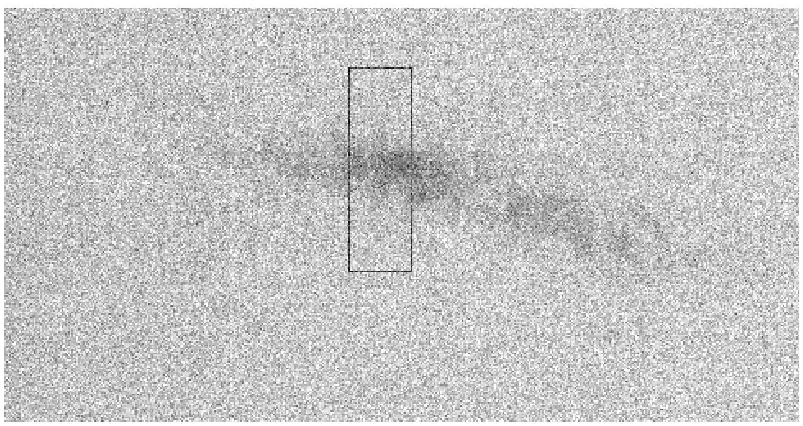

Figure 3. Snapshot of the live streak camera display. The picture's contrast has been altered to improve viewing.

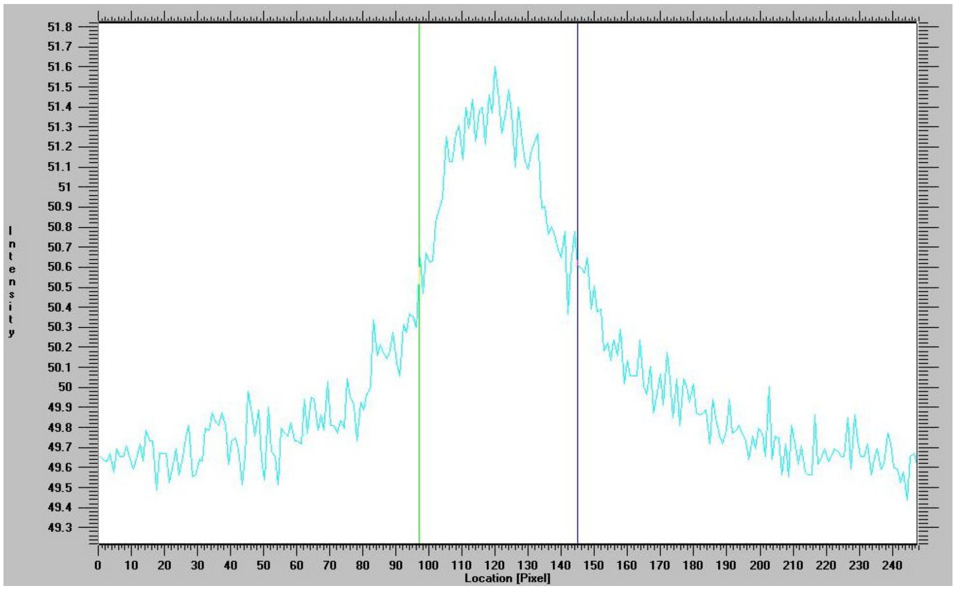

Figure 4. Intensity vs. pixel location graph with a depiction of FWHM.

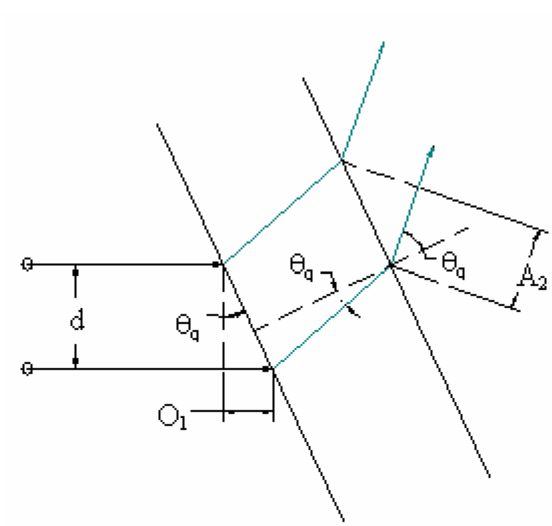

Figure 5.

3. TIME RESOLUTION

An explicit analysis of time resolution can also be done with the given quartz. Figure 5 shows the basic geometry of a pair of electrons passing through the Cherenkov radiator. For this analysis, two electrons separated by a distance d and with the same longitudinal coordinate pass through the radiator. By calculating the distances that the light must travel, a time delay can be found between two photons. The first distance prior to entry is

$$\tan\,\theta_q = O_1/A_1 \Rightarrow O_1 = A_1 \cdot \tan\,\theta_q = d \cdot \tan\,\theta_q = 2.87 \cdot 10^{-4}\ \mathrm{m}.$$

Its hypotenuse,

$H_1 = (d^2 + d^2 \cdot tan\ \theta_q)^{1/2} = d \cdot (1 + tan\ \theta_q)^{1/2}$,

is then used to find the refracted angle of the delayed photon when it exits. Using Snell's law,

$n_i \cdot sin\ \theta_I = n_t \cdot sin\ \theta_t \Rightarrow \theta_t = sin^{-1}[(n_i / n_t) \cdot sin\ \theta_I]$.

But $\theta_I = \theta_C - \theta_q$, so

$\theta_t = sin^{-1}[(n_i / n_t) \cdot sin\ (\theta_C - \theta_q)] = 52.73°$.

With $H_1 = H_2$, $d = 0.001$ m, and $\theta_q = 16°$, the second distance A_2 is

$A_2 = H_2 \cdot cos\ (90 - \theta_t) = (d \cdot (1 + tan\ \theta_q)^{1/2}) \cdot cos\ (90 - \theta_t) =$

$(0.01\ \text{m}) \cdot (1+0.0822)^{1/2} \cdot (0.796) = 8.28 \cdot 10^{-4}$ m.

The length of time it takes for both photons to traverse inside the quartz are identical given that the same Cherenkov angle is used. The total time is then found to be

$\Delta t_1 + \Delta t_2 = (d \cdot tan\ \theta_q)/\beta + A_2/c = 9.56 \cdot 10^{-13}\ \text{s} + 2.76 \cdot 10^{-12}\ \text{s} = 3.71 \cdot 10^{-12}\ \text{s}$.

An alternate equation [3],

$\Delta t = d \cdot n \cdot sin\ \theta_C / c = (d/c) \cdot (n^2 - 1/\beta^2)$,

gives the same solution.

4. RADIATION YIELD

In general, the total energy radiated from the Cherenkov effect [4] is

$$\frac{dW}{dl} = \int_{\beta n > 1} \left(1 - \frac{1}{\beta^2 \cdot n^2}\right) \cdot \omega\, d\omega,$$

but the expression above does not offer any valid result and is infinite since no cut-off frequency has been imposed. An upper limit can be set by realizing in the dispersive nature of the medium, whereby radiation is restricted by $n(\omega) > 1/\beta$, and also considering the finite size of an electron. It can be more practical to calculate the total radiation intensity in terms of the number of photons, within a spectral region defined by wavelengths λ_1 and λ_2, with

$$N = 2\pi\alpha l\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)\cdot\left(1 - \frac{1}{\beta^2 \cdot n^2}\right)$$

where:

- l: is the electron track length
- λ_1, λ_2: are the lower and upper limits of the spectral region.
- α: is the fine structure constant equal to $e^2/\leftarrow c$= 1/137
- n: is the average index refraction of the medium.

If the visible spectrum is used (from $4 \cdot 10^{-7}$ m to $6 \cdot 10^{-7}$ m), along with a track length of 1 mm and n of 1.54, approximately 22 photons will be emitted due to a single electron. Coherent effects due to electron bunching may actually intensify this effect.

5. SUMMARY

Acquiring accurate bunch length measurement of an electron beam can be an arduous task. Slight deviations in the calculations and placement of the Cherenkov radiator and optical transport system can result in a loss of light as well as time spent optimizing and adjusting before and especially during an experiment. Using a radiator with a large index of refraction is beneficial for simplicity of design inside the vacuum chamber but only a fraction of the light cone can be collected due to its inherent geometric constraints. A reduction or loss in light makes the signal-to-background ratio even more important at the streak camera where smaller bunch charges may not achieve sufficient levels of light so that bunch lengths may be measured. One particular result from this experimental set-up found a bunch length of 10 ps (FWHM) from a beam energy range of 6.5 to 7.0 MeV with a 60 nC bunch charge.

6 REFERENCES

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4. J. Jelley, *Cherenkov Radiation*, Perganon Press, New York, 1958.